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EFFECT OF DISPERSION FACTORS ON DECAY REGIMES OF SHORT WAVES IN A GAS-SATURATED POROUS MEDIUM

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A linear analysis of the evolution of small fast-oscillating perturbations in a porous medium saturated with a viscous gas is carried out within a wide range of acoustic Reynolds and Peclet numbers.

The problem of propagation of elastic waves in rocks arises in connection with numerous geophysical applications such as search for and exploration of hydrocarbon deposits, determination of physico-mechanical properties of rocks, active seismoacoustic action with the goal of intensifying hydrocarbon extraction, etc. [1-3]. A theoretical analysis of the wave dynamics of gas-saturated rocks implies developing a mathematical model of a porous medium taking into account rheological and thermodynamic features of solid and fluid phases and mechanisms of their interaction. In the classical Frenkel–Biot–Nikolaevskii approach [4-7], rocks are treated as porous media within a continium approximation of the mechanics of heterogeneous media which makes it possible to develop efficiently models for investigation of the propagation mechanism of waves of various nature. In [8-10], an outgrowth of the approach has been proposed that takes into account additionally viscous stresses in the pore fluid and the liquid bound by the surface of the mineral skeleton. Taking into account dispersion factors (viscous stresses and thermal diffusivity) leads to emergence of dimensionless parameters and inverse acoustic Reynolds and Peclet numbers Re_a^{-1} and Pe_a^{-1} at higher spatial terms in the equations of the model. At large $Re_a >> 1$ and $Pe_a >> 1$, an asymptotic investigation of the model with the use of a multiscale decomposition method is possible. In this case, the Cauchy problem for the original system of equations of laws of mass, momentum, and energy conservation is transformed into the Cauchy problem for nonlinear evolution equations (of the Cortevega–de Vries–Burgers type). An analysis of solutions of evolution equations made it possible to distinguish oscillating regimes of wave propagation which emerge as a result of development of an instability.

As is shown by estimates carried out at characteristic values of parameters of rocks and pore fluids, the range of variations in Re_a and Pe_a appears to be rather wide: from $Re_a >> 1$ and $Pe_a >> 1$ for a highly permeable porous medium saturated with a low-viscosity gas to $Re_a \sim 1$ and $Pe_a \sim 10^1$ at a lowered permeability and increased viscosity. In this connection, it is of certain interest to carry out in the first approximation a linear analysis of propagation regimes of elastic waves in gas-saturated porous media in order to estimate their stability within a wide region of Re_a and Pe_a. In what follows, we carry out a comparative analysis of wavelength dependences of phase velocities and decay coefficients of longitudinal waves of the first and second kind and transverse waves, and an instability regime for longitudinal waves of the first kind is distinguished. The effect of the value of the pore pressure is analyzed. It is shown that a change in the pore pressure leads to appreciable differences in the values of velocity and decay coefficients up to emergence of the phenomenon of "opacity" of the medium for longitudinal waves of the second kind.

Formulation of the Porous Medium Model. We consider a porous medium consisting of a thermoelastically deformable skeleton, a viscous liquid bound by the skeleton surface, and a viscous pore gas. The model of a porous medium proposed in what follows differs from the model [4-7] by taking into account viscous stresses in the free gase phase and bound liquid. In this case the tensor of stresses in the gas phase will be as follows (i, j = 1, 2, 3):

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$$P_{ij} = -p\delta_{ij} + \nu_g\,[\partial v_{gi}/\partial x_j + \partial v_{gj}/\partial x_i - (2/3)(\partial v_{gk}/\partial x_k)\,\delta_{ij}]\,. \tag{1}$$

In what follows, we restrict ourselves to the case of a porous medium saturated with a perfect gas:

$$\rho_g = p/(RT_g)\,,\quad E_g = C_g T_g\,. \tag{2}$$

We will assume that the skeleton of the porous medium and the bound liquid form an effective viscoelastic solid phase that manifests the elastic properties of the skeleton and the viscous properties of the liquid. In this case both the skeleton and the bound liquid have identical velocity, temperature, and pressure. With consideration for viscoelastic properties, the rheological relationship for the solid phase can be presented as follows ($i, j = 1, 2, 3$):

$$\sigma_{ij} = Ke_{kk}\delta_{ij} + 2G\,(e_{ij} - e_{kk}\delta_{ij}/3) + \beta_s Kp\delta_{ij} - \varphi_s KT_s\delta_{ij} +$$
$$+ \alpha m\nu_\alpha\,[\partial v_{si}/\partial x_j + \partial v_{sj}/\partial x_i - (2/3)(\partial v_{sk}/\partial x_k)\,\delta_{ij}]\,. \tag{3}$$

The equations of state and thermodynamic relations for the components of the solid phase are as follows:

$$\rho_\alpha = \rho_{\alpha 0}\,(1 - \beta_\alpha\,(\sigma^s_{kk}/3 - \sigma_0) - \varphi_\alpha\,(T_s - T_{s0}))\,;$$
$$\rho_s = \rho_{s0}\,(1 - \beta_s\,(\sigma^s_{kk}/3 - \sigma_0) - \varphi_s\,(T_s - T_{s0}))\,;$$
$$\rho_s dE_s = \rho_s C_s dT_s + \sigma^s_{ij} de_{ij} + \varphi_s T_s d\sigma^s_{kk}/3\,; \tag{4}$$
$$\rho_\alpha dE_\alpha = \rho_\alpha C_\alpha dT_s - \sigma^s_{kk}/(3\rho_\alpha)\,d\rho_\alpha + \varphi_\alpha T_s d\sigma^s_{kk}/3\,,$$

where the actual stresses in the solid phase are determined by the relationship

$$\sigma^s_{ij} = \sigma_{ij}/(1 - (1 - \alpha)\,m) + P_{ij}\,. \tag{5}$$

We introduce the following dimensionless variables and parameters:

$$x' = x/x_0\,,\ t' = t/t_0\,,\ u' = u/x_0\,,\ \rho' = \rho/\rho_0\,,\ P' = P/K_0\,,\ \sigma' = \sigma/K_0\,,$$
$$T' = T/\theta_0\,,\ \beta' = \beta K_0\,,\ \varphi' = \varphi\theta_0\,,\ K' = K/K_0\,,\ G' = G/K_0\,,\ v' = v/v_0\,,$$
$$E' = E/v_0^2\,,\ \nu' = \nu/(K_0 t_0)\,,\ \lambda' = \lambda\theta_0 t_0/(x_0^2 v_0^2 \rho_0)\,,\ C' = C\theta_0/v_0^2\,,$$
$$R' = R\theta_0/v_0^2\,,\ \chi' = \chi\theta_0 t_0/(v_0^2\rho_0)\,,\ k' = kx_0\,,\ \omega' = \omega t_0\,,$$

where $x_0 = v_0 t_0$; $t_0 = \rho_0\kappa/\nu_g$; $v_0 = (K_0/\rho_0)^{1/2}$.

Dropping primes, we write the system of equations of mass, momentum, and energy conservation in dimensionless form ($i, j = 1, 2, 3$):

$$\partial\,((1 - \alpha)\,m\rho_g)/\partial t + \nabla_x\,((1 - \alpha)\,m\rho_g v_g) = 0\,;$$
$$\partial\,(\alpha m\rho_\alpha + (1 - m)\,\rho_s)/\partial t + \nabla_x\,((\alpha m\rho_\alpha + (1 - m)\rho_s)\,v_s) = 0\,;$$
$$(1 - \alpha)\,m\rho_g\,[\partial/\partial t + \langle v_g, \nabla_x\rangle]\,v_{gi} - (1 - \alpha)m\partial P_{ij}/\partial x_j + m^2\,(1 - \alpha)^2\,(v_{gi} - v_{si}) = 0\,;$$
$$(\alpha m\rho_\alpha + (1 - m)\,\rho_s)\,[\partial/\partial t + \langle v_s, \nabla_x\rangle]\,v_{si} - \partial\sigma_{ij}/\partial x_j -$$

$$-(1-(1-\alpha)m)\,\partial P_{ij}/\partial x_j - m^2(1-\alpha)^2(v_{gi}-v_{si}) = 0;$$

$$m(1-\alpha)\rho_g\,[\partial/\partial t + \langle v_g, \nabla_x\rangle]\,E_g = m(1-\alpha)\,P_{ij}\partial v_{gi}/\partial x_j +$$

$$+ m^2(1-\alpha)^2\,|v_g - v_s|^2 - \chi(T_g - T_s) + \nabla_x m(1-\alpha)\lambda_g\nabla_x T_g;$$

$$(1-m)\rho_s\,[\partial/\partial t + \langle v_s, \nabla_x\rangle]\,E_s + \alpha m\rho_\alpha\,[\partial/\partial t + \langle v_s, \nabla_x\rangle]\,E_\alpha = [\sigma_{ij} +$$

$$+ (1-(1-\alpha)m)\,P_{ij}]\,\partial v_{si}/\partial x_j + \chi(T_g - T_s) + \nabla_x((1-m)\lambda_s + \alpha m\lambda_\alpha)\nabla_x T_s;$$

$$\partial u_i/\partial t - v_{si} = 0;\quad e_{ij} - [\partial u_i/\partial x_j + \partial u_j/\partial x_i]/2 = 0. \tag{6}$$

Relations (1)-(6) constitute a closed system of equations with respect to unknown tensor functions σ_{ij} and e_{ij}, vector functions u, v_s, and v_g, and scalar functions m, p, T_s, and T_g. It contains a number of dimensionless parameters: k, G, β_s, β_α, φ_s, φ_α, etc. Substitution of rheological and thermodynamic relationships into the system of conservation laws leads to inverse Reynolds and Peclet numbers Re_a^{-1} and Pe_a^{-1} at the second spatial derivatives in the equations of momentum and energy conservation, respectively. It is evident that

$$\mathrm{Re}_a = v_0 x_0 \rho_0/\nu_0 \equiv K_0\rho_0\kappa/\nu_0^2,\quad \mathrm{Pe}_a = v_0 x_0\rho_0 C_0/\lambda_0 \equiv K_0\rho_0 C_0\kappa/(\nu_0\lambda_0).$$

Estimates of values of the dimensionless parameters can be obtained with the use of characteristic values of constants of rocks and pore fluids [5]: $K_0 \sim 10^8 - 10^9$ Pa, $\theta_0 \sim 10^2 - 10^3$ K, $\rho_0 \sim 10^1 - 10^3$ kg/m^3, $\beta \sim 10^{-10} - 10^{-9}$ Pa^{-1}, $\varphi \sim 10^{-6} - 10^{-3}$ K^{-1}, $C_0 \sim 10^3$ J/(kg·K), $\lambda_0 \sim 10^{-3} - 10^0$ W/(m·K) , $\nu_0 \sim 10^{-5} - 10^{-4}$ Pa·sec, and $\kappa \sim 10^{-15} - 10^{-7}$ m^2. With these values of parameters we have: $t_0 \sim 10^{-9} - 10^{-6}$ sec, $v_0 \sim 10^3 - 10^3$ m/sec, $x_0 \sim 10^{-6} - 10^{-3}$ m, which determines the ranges of variation of $\mathrm{Re}_a \sim 10^0 - 10^6$ and $\mathrm{Pe}_a \sim 10^1 - 10^6$. Thus, in the actual range of parameters of gas-saturated rocks, the effect of dispersion factors (viscous stresses and thermal diffusivity) can be both small (in a high-permeability medium saturated with a low-viscosity gas) and considerable (in a low-permeability medium saturated with a high-viscosity gas).

It should be noted that the chosen method of introduction of dimensionless variables determines the velocity scale v_0 as the characteristic velocity of propagation of elastic perturbations and the time scale t_0 as the characteristic relaxation time of the interphase momentum exchange (interphase viscous friction) and makes it possible to find the corresponding characteristic spatial scale x_0. The obtained quantitative estimates of t_0 and x_0 restrict the region of applicability of estimates of Re_a and Pe_a to the range of high-frequency (short) waves.

Evolution of Small Free Oscillations. In order to investigate regularities of wave propagation in a saturated porous medium, we restrict ourselves in what follows to the Cauchy problem with the initial conditions ($i, j = 1, 2, 3$)

$$u_i|_{t=0} = u_i^0,\quad v_{si}|_{t=0} = v_{si}^0,\quad v_{gi}|_{t=0} = v_{gi}^0,\quad m|_{t=0} = m^0,$$

$$p|_{t=0} = p^0,\quad T_g|_{t=0} = T_g^0,\quad T_s|_{t=0} = T_s^0;$$

$$e_{ij}|_{t=0} \equiv e_{ij}^0 = [\partial u_i^0/\partial x_j + \partial u_j^0/\partial x_i]/2;$$

$$\sigma_{ij}|_{t=0} \equiv \sigma_{ij}^0 = Ke_{kk}^0\delta_{ij} + 2G(e_{ij}^0 - (1/3)\,e_{kk}^0\delta_{ij}) + \beta_s K p^0\delta_{ij} -$$

$$-\varphi_s K T_s^0\delta_{ij} + \alpha m^0\nu_\alpha\,[\partial v_{si}^0/\partial x_j + \partial v_{sj}^0/\partial x_i - (2/3)(\partial v_{sk}^0/\partial x_k)\,\delta_{ij}]. \tag{7}$$

We will seek a linear solution of the problem (1)-(7) in the form of a superposition of a slow background motion and its small fast-oscillating perturbation

$$U = U_{ph}(x, t) + \varepsilon U_1 \exp(iS/\varepsilon) . \quad (8)$$

Here ε is a small parameter, $U = (m, v_{gi}, v_{si}, p, T_g, T_s, u_i, \sigma_{ij}, e_{ij})$ is the vector function of sought quantities, and the phase $S/\varepsilon = \langle k\xi \rangle - \omega\tau$, where $\tau = t/\varepsilon$ and $\xi_i = x_i/\varepsilon$ are fast variables.

For simplicity, we choose as a background solution a homogeneous stationary state U_0 whose existence is determined by the conditions of zero velocities of phase motion ($v_{si0} = v_{gi0} = 0$), identity of temperatures ($T_{g0} = T_{s0} = T_0$), and independence of the constants m_0, p_0, T_0, and u_{i0} on x.

The system of equations (1)-(6) linearized on the background U_0 can be presented in operator form

$$A \cdot U = 0 , \quad (9)$$

where

$$A(\partial/\partial t, \partial/\partial x_i, \partial^2/\partial x_i \partial x_j, U_0) = A_1(\partial/\partial t, \partial/\partial x_i, U_0) +$$

$$+ A_2(\partial^2/\partial x_i \partial x_j, U_0) + A_3(U_0) .$$

Let U_1 be determined by the amplitude of the original perturbation, i.e., by the oscillating portion of the initial conditions $U^0 = (m^0, v_{gi}^0, v_{si}^0, p^0, T_g^0, T_s^0, u_i^0, \sigma_{ij}^0, e_{ij}^0)$, the components of the wave-vector are positive numbers, and frequencies ω can be complex. This approximation corresponds to the Cauchy problem of the evolution of k-waves, which is an analog of free oscillations [11]. In this case the imaginary part of the frequency ω determines the decay coefficient of the wave.

Substitution of the solution in the form (8) into linear system (9) leads to the condition of nontrivial solvability – to a dispersion equation relating ω and k. When analyzing mechanisms of wave propagation in saturated porous media, it is of interest to investigate dependences of phase velocities $V(k) = \mathrm{Re}(\omega/|k|)$ and decay coefficients $\delta(k) = \mathrm{Im}(\omega)$ for various types of waves on various scales of the Reynolds and Peclet numbers and the related small parameter ε. In what follows, we present results obtained under the assumption that the Froude number $\mathrm{Fr} = 1$ for three cases. It should be noted that derivation of dependences of wave characteristics on $|k|$ appears to be possible as a result of consideration of the isotropic stressed-deformed background state. In the case of anisotropy, an analysis depending on the direction of the wave-vector is required.

Analysis of Dispersion Relations. *Case 1*: $\varepsilon^2 = \mathrm{Re}_a^{-1} = \mathrm{Pe}_a^{-1} \sim 10^{-6}$. It is easily seen that substitution of (8) into (9) yields

$$[A_1(\partial/\partial\tau, \partial/\partial\xi_i, U_0) + \varepsilon A_2(\partial^2/\partial\xi_i\partial\xi_j, U_0) + \varepsilon A_3(U_0)] \cdot U_1 = 0 .$$

Therefore, with the accuracy of $O(\varepsilon)$ we have

$$A_1(\partial/\partial\tau, \partial/\partial\xi_i, U_0) \cdot U_1 = 0 .$$

The corresponding dispersion relation is obtained in the form

$$J_1 = \mathrm{Det}\, A_1(-i\omega, ik_i, U_0) = 0 . \quad (10)$$

Here (and in what follows) we do not present the dispersion equation in explicit form, since it is extremely cumbersome. Application of means of computer algebra makes it possible to present (10) as

$$J_1 = \mathcal{P}_4 \mathcal{P}^2 \omega^{17} ,$$

where the roots of the fourth-degree polynomial $\mathcal{P}_4$ determine the frequencies of the direct and reverse longitudinal waves of the first (pressure waves) and second kind (repacking waves), and the roots of polynomial $\mathcal{P}^2$ determine the multiple frequencies of transverse waves with different polarizations. The following investigation is restricted to a numerical analysis with graphical presentation of the results.

In the case under consideration, the system of equations of the model is hyperbolic, and dissipative and dispersion effects are not included in the linear analysis. This leads to the absence of decay ($\mathrm{Im}(\omega) = 0$) and observation of constant values of velocities of all types of waves for all values of k. It should be noted that the linear analysis is the first stage of application of the method of multiscale decompositions for construction of an asymptotic solution with higher accuracy [8-10]. Thus, construction of a solution with the accuracy of $O(\varepsilon^2)$ leads to a Burgers-type nonlinear equation, and the next step of the decomposition (with the accuracy of $O(\varepsilon^3)$) leads to a Cortevega–de Vries–Burgers-type equation. These nonlinear evolution equations already take into account effects of weak dissipation and dispersion and make it possible to analyze decay regimes for waves and their resonance interaction.

Case 2. $\varepsilon = \mathrm{Re}_a^{-1} = \mathrm{Pe}_a^{-1} \sim 10^{-3}$. It is easily seen that substitution of (8) into (9) yields

$$[A_1(\partial/\partial\tau, \partial/\partial\xi_i, U_0) + A_2(\partial^2/\partial\xi_i\partial\xi_j, U_0) + \varepsilon A_3(U_0)]\cdot U_1 = 0,$$

$$[A_1(\partial/\partial\tau, \partial/\partial\xi_i, U_0) + A_2(\partial^2/\partial\xi_i\partial\xi_j, U_0)]\cdot U_1 = 0.$$

The corresponding dispersion equation leads to

$$J_2 = \mathrm{Det}\,[A_1(-i\omega, ik_i, U_0) + A_2(i^2k_ik_j, U_0)] = 0. \tag{11}$$

As is evident from (11), the characteristic equation in the case under consideration accounts for dispersion factors and does not take into account dissipation due to interphase friction. The system of equations of the model is not hyperbolic any more, which is expressed, in particular, in the fact that the frequencies of all selected wave types are complex-valued. Here, estimation of the dependence of phase velocities of waves V (velocity dispersion) and decay coefficients δ on the wavelength in a linear approximation is of definite interest. An asymptotic analysis of this case in a nonlinear formulation implies application of the modified multiscale decomposition method [12] for the case of "strong" dispersion, which falls outside the scope of the present work.

Case 3: $\varepsilon = \mathrm{Re}_a^{-1} = \mathrm{Pe}_a^{-1} \sim 1$. In this case all the factors (inertial, dissipative, and dispersion) appear to be of the same order, and the system of equations of the model contains no small parameter determining the scale of fast variables ($\tau \equiv t$, $\xi_i \equiv x_i$). To analyze the evolution of small perturbations of the background solution we assume additionally that $U_1 \ll U_b$. Then substitution of (8) into (9) yields

$$[A_1(\partial/\partial\tau, \partial/\partial\xi_i, U_0) + A_2(\partial^2/\partial\xi_i\partial\xi_j, U_0) + A_3(U_0)]\cdot U_1 = 0.$$

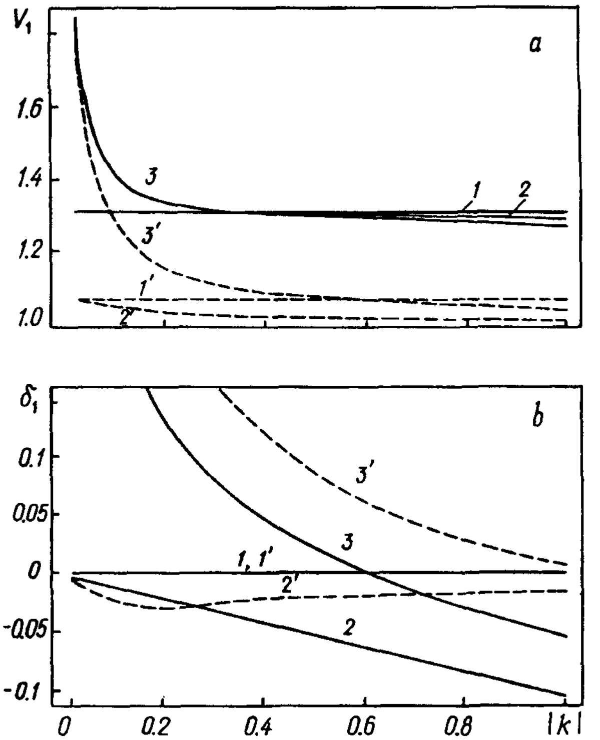
The corresponding dispersion relation has the form

$$J_3 = \mathrm{Det}\,[A_1(-i\omega, ik_i, U_0) + A_2(i^2k_ik_j, U_0) + A_3(U_0)] = 0. \tag{12}$$

In the case under consideration, estimation of the joint effect of dissipative and dispersion mechanisms appears to be possible.

Examples of Calculations. Figures 1-3 present results of calculations of phase velocities V_i and decay coefficients δ_i for longitudinal waves of the first ($i = 1$) and second ($i = 2$) kind and transverse waves ($i = 3$) based on numerical evaluation of roots of dispersion equations (10)-(12). The numbers of the curves presented correspond to the above-considered cases. The calculations were carried out for the following values of the dimensionless parameters: $\nu_g = 0.1$, $\nu_\alpha = 1$, $\rho_{\alpha 0} = 1$, $\rho_{s0} = 2.5$, $\varphi_\alpha = 0.5$, $\beta_\alpha = 0.5$, $\varphi_s = 0.1$, $\beta_s = 0.1$, $G = 0.8$, $K = 1$, $C_\alpha = 2$, $C_g = 1$, $C_s = 1.2$, $\lambda_g = 0.001$, $\lambda_\alpha = 1$, $\lambda_s = 2$, $\chi = 0.1$, $\alpha = 0.1$, $m^0 = 0.2$, $T^0 = 1$, and $e_{ij}^0 = 0$. The solid curves (1, 2, and 3) were obtained for $p^0 = 0.1$, and the dashed curves (1$'$, 2$'$, and 3$'$) correspond to $p^0 = 0.01$.

First, let us consider longitudinal waves of the first kind (Fig. 1a). Case 1 (curves 1 and 1$'$) is characterized by a wavelength-independent phase velocity and the absence of decay (this also holds for all types of waves of Case



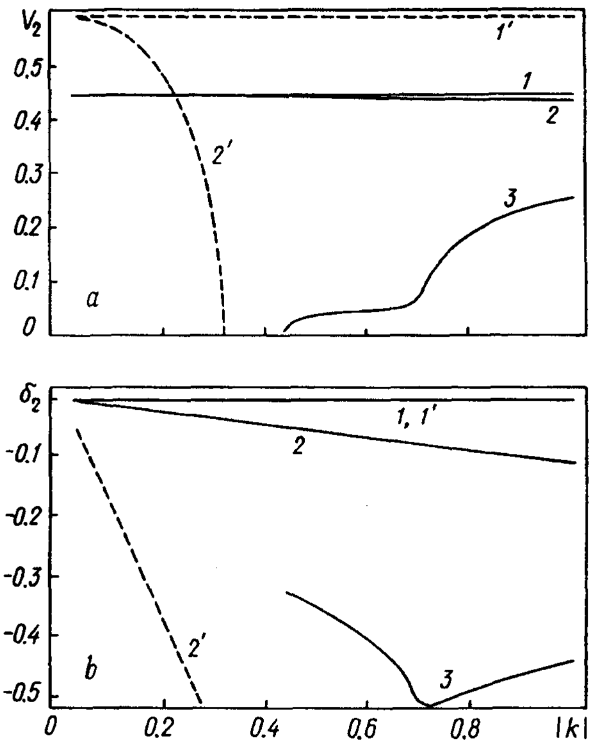


Fig. 1. Plots of phase velocity V_1 (a) and decay coefficient δ_1 (b) versus wavelength for longitudinal waves of the first kind.

Fig. 2. Plots of phase velocity V_2 (a) and decay coefficient δ_2 (b) versus wavelength for longitudinal waves of the second kind.

1). Curves 2 and 2′ (Fig. 1a) are indicative of a tendency to a certain decrease in the velocity of the wave with decreasing wavelength. As is evident from curves 3 and 3′ (Fig. 1a), the joint effect of dispersion and dissipative factors leads to an sharp increase in the velocity in the longwave region. Comparison of curves 2 (2′) and 3 (3′) in Fig. 1a shows that a decrease in the pore gas pressure leads to an appreciable decrease in the phase velocity. The dependence of the decay coefficients in Case 2 (curves 2 and 2′ in Fig. 1b) is nonlinear, and they depend substantially on the value of the background gas pressure. It should be noted that a qualitative difference takes place between the dependences of the decay coefficients: the tendency towards a growth in the absolute value of the decay coefficient with decreasing wavelength gives way to a tendency to its decrease. Case 3 is distinguished by the manifestation of an instability effect, when the decay coefficient becomes positive, and its value increases with the wavelength. A decrease in the pore pressure leads to an expansion of the region of wavelengths where the instability takes place. As a whole, an increase in the pore pressure leads to stabilization of longitudinal waves of the first kind.

The effect of the pore pressure level p^0 manifests itself distinctly by the example of longitudinal waves of the second kind, which are distinguished, as is known, by opposed motion of the solid and fluid phases. As is evident from Fig. 2a, in Case 2, a tendency is observed towards a sharp decrease in the phase velocity (curve 2′, Fig. 2a) with decreasing pore gas pressure, and an increase in the absolute value of the decay coefficient (Fig. 2b) with decreasing wavelength to the extent that an "opacity" region appears when $V_2 = 0$. At the same pore gas pressure in Case 3 the medium appears to be nontransparent at an arbitrary $|k|$ (the missing curve 3′ in Fig. 2a corresponds to $V_2 = 0$). With increasing p^0, the medium becomes "transparent" for short waves, and a tendency towards an increase in the phase velocity with decreasing wavelength takes place (curve 3 in Fig. 2a), and an extremum in the decay coefficient is observed (curve 3 in Fig. 2b).

Transverse waves (Fig. 3) are characterized by a virtually constant phase velocity and are independent of the value of the pore pressure (curves 1, 1′, 2, and 2′ in Fig. 3a) except for Case 3, when an increase in the velocity is observed with decreasing wavelength. As is evident from a comparison of curves 1 (1′), 2 (2′, and 3 (3′) in Fig.

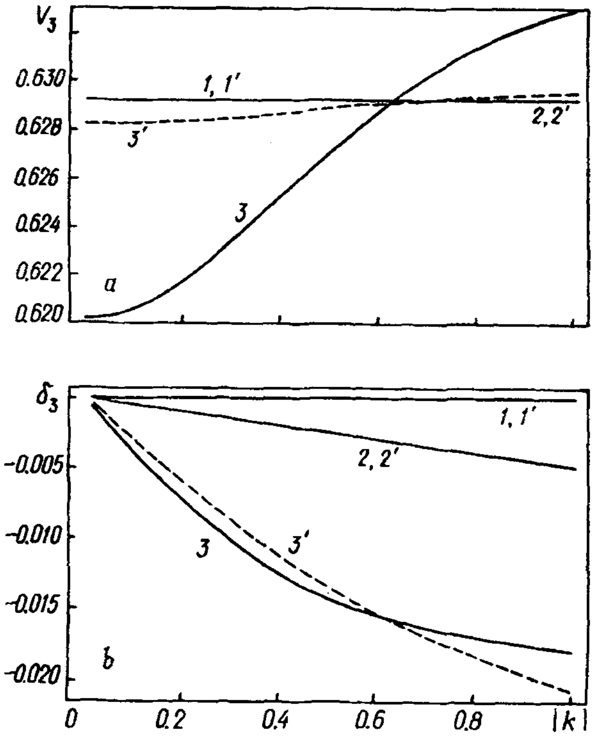


Fig. 3. Plots of dependences of phase velocity V_3 (a) and decay coefficient δ_3 (b) versus wavelength for transverse waves.

3b, decay of the transverse waves increases in the usual manner with an increasing influence of dissipative and dispersion factors.

In conclusion, we emphasize that the linear analysis carried out in the article is not exhaustive, since it is restricted to taking into account only a few parameters. However, it makes it possible to estimate to a first approximation the effect of dispersion and dissipative factors within a wide range of acoustic Reynolds and Peclet numbers and distinguish a number of physical phenomena. In addition, this analysis is a necessary first step towards construction of asymptotic solutions and physical substantiation of the choice of methods for numerical investigation of the model of wave propagation in a gas-saturated porous medium.

NOTATION

x, spatial coordinate; t, time, m, porosity, κ, permeability; α, volume fraction of bound liquid; ρ, density; P_{ij}, tensor of stresses in the gas phase; σ^{s}_{ij}, tensor of actual stresses in the solid phase; σ_{ij}, tensor of effective stresses; v, velocity vector; p, pressure; T, temperature; u, displacement vector; e_{ij}, deformation tensor; K, effective modulus of bulk elasticity; G, shear modulus; β, compressibility factor; φ, thermal expansion factor; E, internal energy; ν, viscosity; λ, thermal diffusivity; C, specific heat; R, gas constant; χ, coefficient of interphase heat transfer; k, wave-vector; ω, frequency; Re_a, acoustic Reynolds number; Pe_a, acoustic Peclet number; V, phase velocity; δ, decay coefficient. Subscripts and superscripts: s, solid phase; g, gas; α, bound liquid.

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